

# INTRODUCTION TO OPTIMIZATION SPRING 2022

## CO 250

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### CLASS SCHEDULE

Section	Location	Time	Instructor(s)
CO 250 001 [LEC]	MC 4020	Mondays & Wednesdays 10 a.m. - 11:20 a.m.	Martin Pei mpei@uwaterloo.ca
CO 250 002 [LEC]		Mondays, Wednesdays & Fridays 1:30 p.m. - 2:20 p.m.	Fei Wang f49wang@uwaterloo.ca
CO 250 003 [LEC]	MC 4059	Tuesdays & Thursdays 11:30 a.m. - 12:50 p.m.	Kanstantsin Pashkovich kpashkovich@uwaterloo.ca

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### INSTRUCTOR / TA INFORMATION

## TUTORIALS

Tutorials are scheduled for Fridays 10:30-11:20am. They will take place at M3 1006. For most tutorials, a TA will present problems and solutions related to the materials done in class.

Quizzes will be held during 3 tutorials (see schedule below). They may take place in different classrooms. Information will be posted on LEARN before the quizzes take place.

## OFFICE HOURS

Instructor office hours will be posted on LEARN.

## CONTACT

You should use Piazza for almost all communications with the instructors. Use email only if you have an issue that is of a very personal nature, and please include "CO 250" in the subject of the email.

### COURSE DESCRIPTION

#### *Calendar Description for CO 250*

A broad introduction to the field of optimization, discussing applications, and solution techniques. Mathematical models for real life applications; algorithms; aspects of computational complexity; geometry; linear programming duality, focusing on the development of algorithms. [Offered: F,W,S]

Prereq: One of (MATH 106, MATH 114, MATH 115 with a grade of at least 70%) or MATH 136 or MATH 146; cumulative overall average of at least 60%. Antireq: CO 227, 255

Suppose that the owner of a factory wants to maximize its production for the next 30 days. There is a limit on the resources available. Resources may include raw materials, labour, machine capacities, etc. This is an example of an optimization problem. The function that we are trying to maximize is the *objective function*, and the conditions imposed by the available resources are the *constraints* of the problem. Optimization problems are classified according to the type of objective function and the type of constraints.

One of the simpler mathematical models is the *Linear Programming Problem*, where both the constraints and the objective functions are affine. Even though this may appear at a first glance to be overly restrictive, linear programming algorithms are widely used. Indeed, a survey of Fortune 500 companies shows that 85% of all respondents use such algorithms in their operations. It is not hard however, to imagine applications for which fractional variable values are not desirable. For instance a variable may indicate the number of employees to hire, or a variable may be restricted to values 0 or 1 to indicate one of two possible options (e.g., build a factory in Waterloo or not). In these cases, we would like to add the condition that some variables in our linear programming problem take only integer values. These mathematical models are known as *Integer Programming Problems*. Finally, in certain instances, such as portfolio optimization (in financial mathematics), the natural way of formulating the optimization problem may require the use of nonlinear functions in the constraints or objective function. This leads to the *Nonlinear Programming* model.

Note however, that the applications of Optimization are not restricted to problems of business. The same techniques have been very successful in Engineering as well as in Physical and Social Sciences. The theory of optimization is utilized in mathematical research in many other mathematical research areas to lead to new advances, to discover and prove new theorems, and to discover and design new algorithms.

In the first part of the course, we will illustrate these various models with examples that arise from real problems. The later part of the course addresses the subject of how to solve the aforementioned problems. The *Simplex Method* to solve Linear Programming problems will be discussed in some detail. And general-purpose Integer Programming techniques such as *Branch-and-Bound* and *Cutting Planes* will also be described. These algorithms, while guaranteed to terminate, may in the worst case (and often do in practice) take a prohibitively long time. No fast general algorithm is known for integer programs and none is believed to exist. However, there are efficient algorithms for many important special cases such as the *Maximum Weight Matching* problem. An indispensable tool for the design of such fast algorithms is the *Theory of Duality*, which will be a main focus of this course.

In the concluding part of the course, we include a review of the various techniques used to solve linear and integer programs, and provide a geometric interpretation of these algorithms. This discussion leads us to nonlinear convex optimization problems.

## LEARNING OUTCOMES

### By the end of this course students should be able to:

Formulate simple real life problems as linear, integer, or continuous (nonlinear) optimization problems.

Understand and be able to execute all the steps of various algorithms and their variants such as the Simplex Method and its variants; moreover, be able to explain rigorously the derivation as well as all the steps of these algorithms.

Formulate the dual of various linear programming problems.

Understand and be able to explain how duality theory is used to develop the shortest path algorithm; and understand the optimality conditions for linear, integer and nonlinear optimization problems.

Develop the ability to utilize the main proof techniques and logical deduction skills in the course; as well as have the ability of independently proving simple related concepts.

Rigorously explain and apply the geometric interpretation of the various algorithms and theorems covered, such as those involving optimality conditions and infeasibility certificates.

## TENTATIVE COURSE SCHEDULE

Week	Topics	Assessments
1	Introductions, linear program formulations	Assignment 1 due Monday May 9
2	Integer program formulations, graph problems	Assignment 2 due Monday May 16
3	Nonlinear program formulations, outcomes of linear programs	Assignment 3 due Tuesday May 24
4	Standard equality form, basis, canonical forms	Quiz 1 Friday May 27
5	Simplex method	Assignment 4 due Monday June 6
6	Geometry of simplex method, convexity, extreme points	Assignment 5 due Monday June 13
7	Duality theory, weak duality, strong duality	Assignment 6 due Monday June 20
8	Complementary slackness, geometry of optimality, Farkas' Lemma	Quiz 2 Friday June 24
9	Shortest path algorithm	Assignment 7 due Monday July 4
10	Solving integer programs, cutting planes	Assignment 8 due Monday July 11
11	Nonlinear optimization, convex functions	Assignment 9 due Monday July 18
12	KKT theorem	Quiz 3 Friday July 22
13	Review	

## TEXTS / MATERIALS

Title / Name	Notes / Comments	Required
A Gentle Introduction to Optimization, Cambridge University Press, 2014 (B. Guenin, J. Koenemann, and L.Tuncel).		No

## STUDENT ASSESSMENT

Component	Value
Assignments (best 8 out of 9)	20%
Quizzes	45%
Final exam	35%

If you have passing grades for at least 4 assignments and at least 2 quizzes, then additionally your lowest 2 assignments and 1 quiz can be replaced by your final exam grade (provided that the final exam grade is higher). The purpose of this is to accommodate any circumstances you may encounter during the term, including (but not limited to) illness, events, internet problems, and technical issues.

If you miss an assignment or a quiz for any reason, do not contact the instructors, the grading scheme has already accommodated for it. You are expected to put full effort into every assignment and quiz. Then you are covered if you encounter unexpected situations and are forced to skip one.

## ASSIGNMENTS

There are 9 assignments in this course. They are usually due on Mondays at 11:59pm EDT (see schedule above). We will be using the Crowdmark system for submitting assignment problems online. Your submission must be clear and legible, and in the correct orientation. You need to submit your solutions to the correct problems, otherwise it will not be marked. Late submissions will receive a mark deduction of 5% for every (full or partial) hour late (up to 100%).

You are expected to do the assignments on your own. The only sources allowed for doing the assignments are:

- all of the material on the CO 250 Spring 2022 course website,
- the textbook,
- all of the material on the CO 250 Spring 2022 Piazza website,
- your Instructors and TAs.

Usage of any other source (dead or alive) in doing the homework assignments is against the academic integrity rules for CO 250 in the Spring 2022 term.

Teaching staff for the course actively look for evidence of academic offences when evaluating assignment submissions. By uWaterloo policy, academic integrity violations by a student in assignments will result in a mark of zero in that assignment and potentially will lead to a 5% deduction from the Final Course Grade. In addition, all academic offences are reported to the Associate Dean for Undergraduate Studies and are recorded in the student's file (this may lead to further, more severe consequences).

A missed assignment will be treated the same as a mark of zero.

## QUIZZES

There are 3 quizzes in this course. They take place during tutorials (see schedule above). These are closed-book exams. No aids are allowed, except for one letter-size reference sheet that you have prepared on your own.

A missed quiz will be treated the same as a mark of zero.

## FINAL EXAM

The final exam will be comprehensive, and is scheduled by the Office of the Registrar later in the term. This is a closed-book exam. No aids are allowed, except for one letter-size reference sheet that you have prepared on your own.

## ASSIGNMENT SCREENING

No assignment screening will be used in this course.

## NOTICE OF RECORDING

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Activities for this course involve recording, in partial fulfillment of the course learning outcomes. You will receive notification of recording via at least one of the following mechanisms: within the Learning Management System (LEARN), a message from your course instructor, course syllabus/website, or other means. Some technologies may also provide a recording indicator. Images, audio, text/chat messaging that have been recorded may be used and/or made available by the University to CO 250 students and staff for the purpose of reviewing course materials. Recordings will be managed according to the University records classification scheme, [WatClass](https://uwaterloo.ca/records-management/records-classification-and-retention-schedules) (<https://uwaterloo.ca/records-management/records-classification-and-retention-schedules>), and will be securely destroyed when no longer needed by the University. Your personal information is protected in accordance with the [Freedom of Information and Protection of Privacy Act](https://www.ontario.ca/laws/statute/90f31) (<https://www.ontario.ca/laws/statute/90f31>), as well as [University policies and guidelines](https://uwaterloo.ca/privacy/) (<https://uwaterloo.ca/privacy/>) and may be subject to disclosure where required by law.

The University will use reasonable means to protect the security and confidentiality of the recorded information, but cannot provide a guarantee of such due to factors beyond the University's control, such as recordings being forwarded, copied, intercepted, circulated, disclosed, or stored without the University's knowledge or permission or the introduction of malware into computer system which could potentially damage or disrupt the computer, networks, and security settings. The University is not responsible for connectivity/technical difficulties or loss of data associated with your hardware, software or Internet connection.

By engaging in course activities that involve recording, you are consenting to the use of your appearance, image, text/chat messaging, and voice and/or likeness in the manner and under the conditions specified herein. (In the case of a live stream event, if you choose not to have your image or audio recorded, you may [disable the audio and video functionality](https://uwaterloo.ca/student-it-services/) (<https://uwaterloo.ca/student-it-services/>). Instructions to participate using a pseudonym instead of your real name are included where the feature exists; however, you must disclose the pseudonym to your instructor in advance in order to facilitate class participation.) If you choose not to be recorded, this notice serves as confirmation of your understanding that you may view the recording later on your own.

You are not permitted to disclose the link to/URL of an event or an event session recording or copies of recording to anyone, for any reason. Recordings are available only to authorized individuals who have been directly provided the above instructions/link for their use. Recordings for personal use, required to facilitate your learning and preparation of personal course/lecture notes, should not be shared with others without the permission of the instructor or event coordinator. Review the University's [guidelines for faculty, staff and students entering relationships with external organizations offering access to course materials](https://uwaterloo.ca/secretariat/faculty-staff-and-students-entering-relationships-external) (<https://uwaterloo.ca/secretariat/faculty-staff-and-students-entering-relationships-external>) for more information on your obligations with respect to keeping copies of course materials. For more information about accessibility, connect with [AccessAbility Services](https://uwaterloo.ca/accessability-services/) (<https://uwaterloo.ca/accessability-services/>).

## ADMINISTRATIVE POLICY

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### PIAZZA

We will be using Piazza to facilitate discussions during this course. The course staff will visit Piazza on a regular basis and will try to answer questions in a timely manner. Some general guidelines on Piazza posting:

- The aim is to create a positive learning environment for everyone. Please be professional and courteous. (Though an occasional meme post might be ok.)
- For assignment problems, you can ask for clarifications, but do not post any full or partial solutions, nor any strong hints. If you have a specific question that include partial solutions to assignment problems, please use private posts.
- Please put some thoughts into your questions and posts. You should show that you have worked on the problem in some way. A post similar to "I'm stuck, any hints?" is not appropriate and will not receive help.

- You are encouraged to help each other and answer questions. For student answers that are correct and do not require additional clarification, the course staff will endorse them without making a further response.
- Often times the course staff do not give you a direct response to your question. Instead, we may ask a probing question that helps you discover or verify the answer on your own.

## RE-MARKING POLICIES

**For assignments:** If you believe that there are marking errors in your assignments and you would like to make a re-marking request, then you need to create a private Piazza post that includes your student ID and which assignment problem or which quiz that you want re-marked, with an explanation. You need to do this within one week of the date the assignment is marked and returned to you.

Note that your solution will be very carefully examined during a re-mark. So it is possible that your grade is reduced. Also, we cannot mark anything that is not in your submission. For example, if your solution has some missing details and you try to provide them in your request, we will not consider that. Moreover, complaints about the marking scheme itself is not a valid reason for adjusting marks. For example, say you think that one type of mistake leads to too many marks being taken off. As long as this is consistently applied to all students, then it is not a marking error.

**For quizzes:** Re-marking requests for quizzes can only be done while visiting your instructor during their office hours.

## INC POLICY

In case of serious illness during the final exam, you may be granted a grade of INC provided that

- you have suitable medical documentation that you submit to the Math Undergraduate Office; and
- you have passing grades for at least 4 assignments and at least 2 quizzes.

Granting a grade of INC is at the discretion of your instructor.

## INTELLECTUAL PROPERTY

Students should be aware that this course contains the intellectual property of their instructor, TA, and/or the University of Waterloo.

Intellectual property includes items such as:

- Lecture content, spoken and written (and any audio/video recording thereof);
- Lecture handouts, presentations, and other materials prepared for the course (e.g., PowerPoint slides);
- Questions or solution sets from various types of assessments (e.g., assignments, quizzes, tests, final exams); and
- Work protected by copyright (e.g., any work authored by the instructor or TA or used by the instructor or TA with permission of the copyright owner).

Course materials and the intellectual property contained therein, are used to enhance a student's educational experience. However, sharing this intellectual property without the intellectual property owner's permission is a violation of intellectual property rights. For this reason, it is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online (e.g., to an online repository).

Permission from an instructor, TA or the University is also necessary before sharing the intellectual property of others from completed courses with students taking the same/similar courses in subsequent terms/years. In many cases, instructors might be happy to allow distribution of certain materials. However, doing so without expressed permission is considered a violation of intellectual property rights.

Please alert the instructor if you become aware of intellectual property belonging to others (past or present) circulating, either through the student body or online. The intellectual property rights owner deserves to know (and may have already given their consent).

## UNIVERSITY POLICY

**Academic integrity:** In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check [the Office of Academic Integrity](https://uwaterloo.ca/academic-integrity/) (<https://uwaterloo.ca/academic-integrity/>) for more information.]

**Grievance:** A student who believes that a decision affecting some aspect of their university life has been unfair or unreasonable may have grounds for initiating a grievance. Read [Policy 70, Student Petitions and Grievances, Section 4](https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70) (<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70>) . When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for their actions. [Check [the Office of Academic Integrity](https://uwaterloo.ca/academic-integrity/) (<https://uwaterloo.ca/academic-integrity/>) for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to [Policy 71, Student Discipline](https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71) (<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71>) . For typical penalties, check [Guidelines for the Assessment of Penalties](https://uwaterloo.ca/secretariat/guidelines/guidelines-assessment-penalties) (<https://uwaterloo.ca/secretariat/guidelines/guidelines-assessment-penalties>) .

**Appeals:** A decision made or penalty imposed under [Policy 70, Student Petitions and Grievances](https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70) (<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70>) (other than a petition) or [Policy 71, Student Discipline](https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71) (<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71>) may be appealed if there is a ground. A student who believes they have a ground for an appeal should refer to [Policy 72, Student Appeals](https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-72) (<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-72>) .

**Note for students with disabilities:** [AccessAbility Services](https://uwaterloo.ca/disability-services/) (<https://uwaterloo.ca/disability-services/>) , located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

**Turnitin.com:** Text matching software (Turnitin®) may be used to screen assignments in this course. Turnitin® is used to verify that all materials and sources in assignments are documented. Students' submissions are stored on a U.S. server, therefore students must be given an alternative (e.g., scaffolded assignment or annotated bibliography), if they are concerned about their privacy and/or security. Students will be given due notice, in the first week of the term and/or at the time assignment details are provided, about arrangements and alternatives for the use of Turnitin in this course.

It is the responsibility of the student to notify the instructor if they, in the first week of term or at the time assignment details are provided, wish to submit alternate assignment.